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Determination of Generator Participation in Loads and Lines using Extended Incidence Matrix method

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Abstract— In the deregulated environment generation companies and other utilities become the user of transmission services. So, there is growing need to identify the participation of each generators and cost of transmission services. In such situation dispatchers not only concentrate on the power flow of the system but also contribution of each generator to each load and losses allocated to the generators. In this paper a mathematical method is proposed for the active power flow tracing based on the concept of extended incidence matrix. By obtaining the AC or DC load flow solution, extended incidence matrix, distribution factor matrix and load extraction matrix are derived. With the help of developed model power transfer between each generator to each load, each line and contribution of each generator to the system losses is calculated. The effectiveness of proposed method is illustrated using IEEE 4-bus and 6-bus systems.

Index Terms— Active power flow, deregulation, distribution factor matrix, extended incidence matrix, generator participation, gross power flow tracing, independent power producer.

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1. Introduction:

IN the open access environment transmission become special business that provides wheeling charges to the Independent power producers (IPPs) and others utilities. IPPs and utilities becomes the user of transmission services. There for, there is growing need to determine the contribution of generators and cost of transmission services. In this case it is very important to calculate individual generators contribution to the load and losses to the system [1,10]. Active and reactive tracing of power using imaginary current is proposed [8].In the recent years many papers have contributed on these topics. Contribution of individual generators to load flow using proportional sharing assumptions and condition for maximum flow through the line proposed [3]. A graph theory is proposed to calculate contribution factor of individual generators to line and load extraction factor of individual load is presented [2]. Upstreaming and down streaming algorithm for the power flow tracing in ref [1] and with change in load flow, generators participating factor method [5].

In most of proposed methods, proportional sharing principle is used power flow tracing is published which described that contribution of inflow (a line flow entering node) to each out flow is in same proportion as the inflow on each line divided by total inflow of all line at a node in ref [1,5]. All principle easily utilized from the application point of view. Hoe ever a disadvantage in case of sharing principle for electricity transfer between generators to loads does not give a mathematically strict derivation. In ref[7] presented an optimization for tracing of power flow and transmission fixed cost allocation. A game theory is suggested in [2] for proportional sharing assumption in tracing methodology. The proposed method is a mathematical method based on concept of Extended Incidence Matrix. This method does not need any assumption concerning with principle of power sharing. It can handle the any power system network with loop or without loop and problem formulation is same as that of any AC load flow solution or DC load flow solution. This method is applicable to only loss-less network.

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2. Extended Incidence Matrix

2.1 Formation of loss less power system

An AC load flow solution is obtained by off line program shown in figure 1. The real and reactive power required by transmission line resistance, reactance and capacitance (R-L-C) moved to the loads and modeled as equivalent constant load. There for line active power flow keep constant along the line and with direction and system become loss-less.

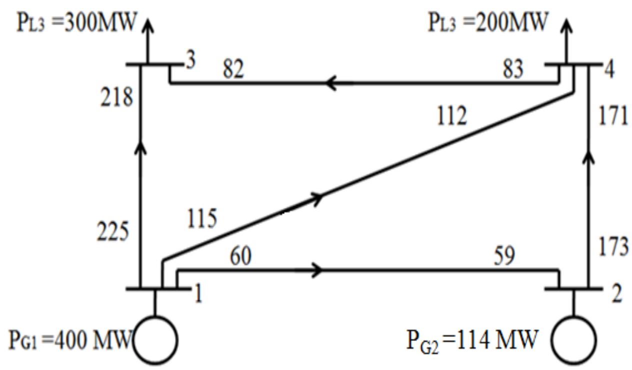

Fig.1 AC real power flow in 4-bus sysytem

A generator has the priority to provide power to the load on the same bus, and the reaming power will enter the network supply other load in the network to avoid unnecessary losses

2.2 Formation of Extended Incidence Matrix

According to Kirchhoff's first low, total inflow at a node equal to total out flow from the same node in any network. Here inflow is defined as sum of power injected by the source and power imported by the other buses. The outflow is defined as the sum of power extracted from a bus by load and power exported to other buses. Diagonal elements give gross flows at busses and off diagonal elements give the actual flows and counter flows in the system An extended incidence matrix (Aij) of a loss less power system is square matrix built by following expression (1).

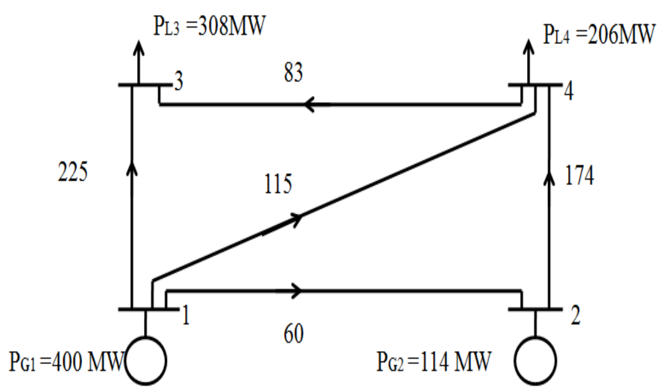

Fig. 2 Loss-less power system

$$A_{ij} = \begin{cases} -P_{ij} & \text{for } i \neq j \text{ and } P_{ij} > 0 \\ 0 & \text{for } i \neq j \text{ and } P_{ji} > 0 \\ P_{Ti} & \text{for } \quad i = j \end{cases}$$

where, (1)

$$P_{Ti} = \sum_{k=1}^{n} P_{ki} + P_{gi} \quad , P_{ki} > 0,$$

$$i,j = 1,2....n$$

The Extended incidence matrix of a loss-less power system shown in figure 2 is given below in equation (2). Matrix A gives relation between direction of active power flow, total power injection at bus and bus to bus incidence relations.

$$A = \begin{bmatrix} 400 & -60 & -225 & -115 \\ 0 & 174 & 0 & -174 \\ 0 & 0 & 308 & 0 \\ 0 & 0 & -83 & 289 \end{bmatrix}_{4\times4} \qquad (2)$$

2.3 Properties of EIM

Property1. Extended incidence matrix is n×n square matrix and is an invertible matrix. Hence A^{-1} is exists.

$$A^{-1} = \begin{bmatrix} 0.0025 & 0.0009 & 0.0022 & 0.0015 \\ 0 & 0.0057 & 0.009 & 0.0035 \\ 0 & 0 & 0.0032 & 0 \\ 0 & 0 & 0.0009 & 0.0035 \end{bmatrix}_{4\times4} \qquad (3)$$

Property2. The sum of all elements in column of k of an EIM equal to total active power of generators at bus k. this property mathematically express as

$$A^{T} E = P_{G} \qquad (4)$$

Vector $[400\ 114\ 0\ 0]^{T}$ is output of generators (PG).

Property3. The sum of all elements in row of k of an EIM equal to total active load power at bus k, as

$$AE = P_{L} \qquad (5)$$

For the system load vector (P_L) is [0 0 308 206].

From equation (4) and (5) , matrix E can be written as

$$E = (A^{-1})^{T} P_{G} \qquad (6)$$

$$E = A^{-1} P_{L} \qquad (7)$$

Where,
E is unity matrix of n×1 dimension.

From equation (1) Extended incidence matrix is built having large number of zero elements in the structure for loss-less power system without loop. In others words extended incidence matrix is widely distributed matrix.

3 Model for Gross Power Flow Tracing of Loss-Less Power System.

3.1 Participation of each generator to each load.

The generator capacity can be represented in a matrix and the diagonal matrix is PGG =diag(PG1 , PG2 ,..... PGn). The capacity of individual generator of the system is given by the expression (7)

$$P_G = P_{GG} \times E \tag{8}$$

From the equation (7) and equation (8)

$$P_G = P_{GG} \times A^{-1} \times P_L \tag{9}$$

Equation (9) gives the relation between P_G and P_L i.e. participation of each generator to each load and is named as distribution factor matrix denoting as D.

$$P_G = D \times P_L \tag{10}$$

$$D = P_{GG} \times A^{-1} \tag{11}$$

In general generators participation is given by

$$P_{Gi} = \sum_{j=1}^{n} D_{ij} P_{Lj} \tag{12}$$

Where $D_{ij}P_{Lj}$ equals the active power of generator output at bus i to the load bus j.

$$D = \begin{bmatrix} 1 & 0.3448 & 0.8937 & 0.6055 \\ 0 & 0.6552 & 0.1063 & 0.3945 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$P_{GG} = \begin{bmatrix} 400 & 0 & 0 & 0 \\ 0 & 114 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad 14)$$

Distribution factor matrix for the fig.2 is given in equation (13). And diagonal matrix for the system is given in equation (14). The sum of all elements in column of distribution factor matrix is unity. This property verifies the correctness to the distribution factor matrix can be applied to any power system.

$$E^T D = E^T \tag{15}$$

Equation (9), (11) and (15) satisfy the power extraction relation and generator participation relation to each load. For the given system power transfer from generator 1 to load 4 and generator 2 to load 3 as

$$P_{1 \to 3} = 0.8937 \times 308 = 275.259 \text{ MW} \tag{16}$$

$$P_{2 \to 4} = 0.3944 \times 206 = 81.267 \text{ MW} \tag{17}$$

3.2 Participation of each generator to line flow.

From the distribution factor matrix, participation of each generator to load from the different transmission line in the network can be obtained. There for the distribution factor from any generator i to line s-t is equal to distribution factor from generator bus i to bus s.
i.e.

$$P_{i \to s-t} = D_{is} \times P_{s \to t} \tag{18}$$

Where,

$P_{s \to t}$ = power flow in each line (s-t)

$P_{i \to s-t}$ = power flow in line (s-t) due to i^{th} generator

$$\begin{aligned} P_{1\to 2-4} &= D_{12} \times P_{2\to 4} \\ &= 0.3448 \times 174 \\ &= 59.9952\text{MW} \end{aligned} \tag{19}$$

$$\begin{aligned} P_{1\to 4-3} &= D_{14} \times P_{4\to 3} \\ &= 0.6055 \times 83 = 50.256 \text{ MW} \end{aligned}$$

$$\begin{aligned} P_{2\to 4-3} &= D_{24} \times P_{1\to 3} \\ &= 0.3945 \times 83 \\ &= 32.743 \text{ MW} \end{aligned}$$

Line 3-4 carry 50.256 MW power of generator 1 and 32.743 MW power of generator 2

3.3 Power extracted from each generator by loads

Section 3.1 and 3.2 described idea about participation of each generator to each load and each line. Power flow tracing in ref [1, 2] comes in two flavors as Up-streaming algorithm and down streaming algorithm. Similarly, tracing of power flow can be done in another way as power extracted from each load from each generator. Taking the dual of equation (9) gives the relation between load vector (PL) and generator output vector (PG). Equation (20) gives transformation of extended incidence matrix

$$P_L = P_{LL} \times (A^{-1})^T \times P_G \tag{20}$$

Rewriting the equation (20)

$$P_L = L \times P_G \tag{21}$$

$$L = P_{LL} \times (A^{-1})^T \tag{22}$$

Where,

L= Load extraction matrix

Similarly, extracted load power can be represented by matrix L and diagonal matrix P_{LL} =diag(PL1,PL2,...PLn). For the figure 2 P_{LL} is given in equation (24).

$$P_{3\leftarrow 1} = 0.6881 \times 400 = 275.24 \text{ MW}$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.6881 & 0.2872 & 1 & 0.2872 \\ 0.3119 & 0.7128 & 0 & 0.7128 \end{bmatrix} \tag{23}$$

$$P_{LL} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 308 & 0 \\ 0 & 0 & 0 & 206 \end{bmatrix} \tag{24}$$

4 Case Studies

A four bus system is as shown in fig. (1) is discussed in detail to compute participation of generator to loads where 1 and 2 are generator busses and 3 and 4 are load busses. Participation of generators to the loads is obtained by extended incidence matrix. For the tracing of power flows based on propose method a program is coded in MATLAB. An example of 6-bus system is also demonstrated by this method.

Table1 Result of EIM Algorithm of Participation of Generators to the Loads

Loads	Sharing load by		Total allocation to the loads (MW)	Losses allocated to Loads (MW)
	G_1	G_2		
L_3	275.2596	32.7404	308	8
L_4	124.733	81.267	206	6
Total	400	114	514	14

5 Results

The gross power flow transfer from generators to loads in the system and the losses allocated to the loads are given in Table 1. The sum of the total power generation at the generators busses is equal to the total demand including losses at the load buses. These results are approximately same as up-streaming algorithm (assuming proportional sharing assumptions).

Table2 Result of EIM Algorithm of Participation of Each Generator to the Line Flow.

Line	Sharing load by		Total (MW)	Losses allocated in line (MW)
	G_1	G_2		
1-2	60	0	60	1
1-3	225	0	225	7
1-4	115	0	115	3
2-4	59.9952	114.0048	174	2
4-3	50.2565	32.7435	83	1

Table 3 Result of EIM Algorithm of Power Extraction by the Load from Each Generator.

Generator	Loads		Gross power generation (MW)	Losses allocated to Loads (MW)
	L_3	L_4		
G_1	275.24	32.7408	307.9808	7.9808
G_2	124.76	81.2592	206.0192	5.0192
Total	400	114	514	14

Table4 Result of EIM Algorithm of Participation of Generators to the Loads

Loads	Sharing load by			Total allocation to the loads (MW)	Losses allocated to Loads (MW)
	G_1	G_1	G_3		
L_4	05.8956	32.7404	0	10.20	0.2
L_5	39.8696	31.8846	30.7758	102.53	2.53
L_6	01.7905	16.8953	32.0442	50.73	0.73
Total	47.5100	53.0300	62.8200	163.46	3.46

In the system losses allocated at L3 and L4 are 8.01 MW and 5.99 MW respectively. In table 2 Contribution of each line to the generators is shown. i.e. a particular line carry amount of power from the both generators G1 and G2. By taking duel of equation (9) power extracted by loads are obtained shown in table 3. From the table 1 and table 2 it is verified that total power generation is equal to power extracted by load including losses. Load flow is obtained by NR-method for 6-bus system and generator participation is calculated by proposed method. Table 4 shows gross power flow tracing and losses allocated to the loads. Contribution of transmission line for power flow and power fed to each line by each generator is shown in table 5.

Table 5 Result of EIM Algorithm of Participation of each Generator to the Line flow

Line	Sharing load by			Total (MW)	Losses allocated in line (MW)
	G_1	G_2	G_3		
1-2	5.62	0	0	5.62	0.4
1-4	12.98	0	0	12.98	0.12
1-5	28.95	0	0	28.95	0.68
2-3	0.17723	1.67277	0	1.85	0
2-4	1.088288	10.27171	0	11.36	0.07
2-5	2.32315	21.92685	0	24.25	0.59
2-6	2.02617	19.12383	0	21.15	0.31
3-5	0.067932	0.651644	24.4404424	25.16	0.76
3-6	0.106677	1.023309	38.380014	39.51	0.32
4-5	8.17292	5.96708	0	14.14	0.41
6-5	0.354059	3.33999	6.335951	10.03	0.1

6 CONCLUSION

Extended incidence matrix method is presented in this paper using a IEEE 4-bus and 6-bus systems for generators participation to various loads and line. Extended incidence matrix does not require any assumption regarding to proportional sharing of power flow. EIM can be constructed directly from the loss-less power system. It can be applicable to AC or DC system with same problem formulation. Proposed extended incidence matrix is highly sparse matrix hence there are large number of zero elements. In this method matrix inverse is required only one time and less calculation required.

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